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Far East Journal of Mathematical Sciences (FJMS)

http://www.pphmj.com
http://dx.doi.org/10.17654/MS101081711
Volume 101, Number 8, 2017, Pages 1711-1720 ISSN: 0972-0871

COBB DOUGLAS FUNCTION FOR SOLVING LINEAR PROGRAMMING TO ANALYZE OPTIMUM INCOME OF LOCAL MIGRANTS

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Abstract

This study attempts to solve linear programming with Cobb Douglas functions to determine the optimum level of farm productions, income of local migrants in South Sulawesi and formulate the optimum uses of resource in crop production. The level of resource usage and economic scale were analyzed by using Cobb Douglas functions. The usage of efficiency of crops production by ratio $(\alpha_j(y^*)p/x_{mi}c_j)$ and optimization of crops was analyzed by linear programming. The estimation result of the resource usage showed a positive and very significant roles on production scale which is the decreasing return to scale. The optimum income increased into 29.20%; 15.38%; and 36.94% at local migrant in four units, respectively.

1. Introduction

The main purpose of research operation in mathematics is to derive

Received: October 27, 2016; Accepted: January 13, 2017
2010 Mathematics Subject Classification: 90C05, 90C08.
Keywords and phrases: Cobb Douglas function, linear-programming, usage efficiency.

solutions of an optimization problem, if it is possible. The solution can also improve previous optimum results from a desirable system. A branch of research operation which is considered to be able finding an optimum or the best solution is the linear programming (LP) first introduced by George B. Dantzig. LP is a method for determining and finding solutions of linear programming problems with many variables. A problem in the general application of linear programming is allocating the limited resources among competing activities in a systematic and efficient way [2].

Cobb Douglas function is a nonlinear function which widely used in determining the optimum level of production, income level, efficiency, etc., under competing activities. In this research, Cobb Douglas function is used for determining the usage efficiency of limited resource and finding out its relationship with linear programming. More precisely, linear programming was used in determining the optimum level of income of local migrants in South Sulawesi under limited resources.

One such research had been done in four districts of local migrants. The local migrants' income level was very low due to the fact that the resources owned by local migrant have been not optimum yet [7]. Cobb Douglas analysis is used to determine the efficient level of the resource usages (land, fertilizer, meds, seed, tool, pesticide, capital and labour) from the increase of the local migrant income expenditures. Farming pattern can be determined by combination between food crops (rice, corns, beans) and plantation crops (cashew nuts, cocoas) [1].

The income satisfaction is the optimal idealistic result which is in turn would give a maximum positive impact to the local migrants. Furthermore, sensitivity analysis (post optimal) will be done if any changes occur affecting resource cost.

2. Literature Review

Input and output correlation

Technically, the relationships between input and output states in production function is formulated with a mathematics equation, namely

$y = f(x_1, x_2, ..., x_n)$, where y is its output resulted from the usage inputs $x_1, x_2, ..., x_n$. One of algebraic form of production function is the Cobb Douglas form. This production function requires that amounts of the inputs $x_i s$ and the output y should be relatively balanced. From the first derivative $\frac{\partial y}{\partial x}$, adding an output will ultimately cause the decrease of a result. This due to the fact that the Cobb Douglas has negative second derivative $\left(\frac{\partial^2 y}{\partial x_i^2} < 0\right)$ which shows that ultimately y is decreasing. Therefore, a condition with input addition proportionally causes the decrease of the increasing output [6]. This means

$$\left(\frac{\partial y}{\partial x_i}\right)\left(\frac{x_i}{y}\right) = 1.$$

The Cobb Douglas function is stated as:

$$y_j = \prod_{j=1}^{m} a_0 x_{ij}^{\alpha j} e_j^u, \tag{1}$$

where y_j = income produced from jth plant pattern, a_0 = resource coefficient, $x_{i,j}$ = the area of j-plant pattern of the i-migrant, α_j = elasticity (production scale) of jth plant pattern, e_j^u = error of j-plant measurement.

Marginal product of production factor x_{ij} is:

$$\frac{\partial y_j}{\partial x_{1i}} = a_0 \alpha_1 x_{1i}^{\alpha_1 - 1} x_{2i}^{\alpha_2} \cdots x_{mi}^{\alpha_m},$$

$$\cdots$$

$$\frac{\partial y_j}{\partial x_{1i}} = a_0 \alpha_m x_{1i}^{\alpha_1} x_{2i}^{\alpha_2} \cdots x_{mi}^{\alpha_m - 1}. \tag{2}$$

With some obvious notation conversions, the last equation can be rewritten as

$$\frac{\partial y_j}{\partial x_{mi}} = \frac{\alpha_j(y^*)}{x_{mi}^*}, \tag{3}$$

y^* is an average geometric production and x_{mi}^* is a geometric average from factor j production total. Production using factor would be efficient if the marginal production value equal to the production cost.

Therefore, mathematic form can be written as:

$$P\left(\frac{\alpha_j(y^*)}{x_{mi}^*}\right) = c_j \tag{4}$$

with P is production cost per unit and c_j is production factor cost per unit [3].

From (4), equation can be rewritten as:

$$\left(\frac{\alpha_j(y^*)}{x_{mi}^*}\right)\frac{P}{c_j} = 1. \tag{5}$$

The Cobb Douglas model is based on assumptions on the following scale elasticity categories α:

(a) if $\sum \alpha_j > 1,$ production scale will be on increasing return to scale position,

(b) if $\sum \alpha_j = 1,$ production scale will be on the constant return to scale position,

(c) if $\sum \alpha_j < 1,$ production scale will be on the decreasing return to scale position.

Relationships of Cobb Douglas function and linear programming

The y-isoquant is the set of all sequences $(x_{1i}, x_{2i}, ..., x_{mi}),$ $i = 1, 2, ..., n$ such that

$$F = a_0 x_{1i}^{\alpha_1} x_{2i}^{\alpha_2} \cdots x_{mi}^{\alpha_m}.$$

Frontier production function (FPF) is a production function which is used on measuring position between production function and its frontier position on the y-isoquant lines [7].

The Cobb Douglas function model:

$$y_i = a_0 \sum_{j=1}^{m} x_{ij}^{\alpha_j} e_i^u \tag{6}$$

and value of y_i can be found by taking its logarithm

$$\ln y_i = \ln a_0 + \sum_{j=1}^{m} \alpha_j \ln x_{ij} + \ln e_i^u$$

or

$$y_i = \sum_{j=1}^{m} \alpha_j x_{ij} + u. \tag{7}$$

If equation (7) is estimated with the frontier, u has to be minimized with condition $\hat{y}_i = \sum_{j=1}^{m} a_j x_{ij}$ with $\hat{y}_i \geq y_i$ and $a_j \geq 0$. Therefore this problem is a matter of linear programming where the a_j's is countable.

By summing the perceived sample from equation (7) can also be rewritten:

$$\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} \sum_{j=1}^{m} \alpha_j x_{ij} - \sum_{i=1}^{n} u$$

and

$$\sum_{i=1}^{n} u = \sum_{i=1}^{n} \sum_{j=1}^{m} \alpha_j x_{ij} - \sum_{i=1}^{n} y_i \tag{8}$$

and minimized $\sum_{j=1}^{m} a_j x_{ij}$ with condition [5] $\sum_{j=1}^{m} a_j x_{ij} \geq y_i$.

Therefore, the equation becomes,

$$\text{minimizing: } a_1x_1 + a_2x_2 + \cdots + a_mx_n$$

with the conditions

$$a_1x_{11} + a_2x_{12} + \cdots + a_mx_{1m} \geq y_1$$

$$\cdots$$

$$a_1x_{n1} + a_2x_{n2} + \cdots + a_mx_{nm} \geq y_n.$$

The linear programming analysis is supported by five basic assumptions which are the power of this analysis: (1) linearity, (2) proportionality, (3) addictive, (4) divisibility, (5) determinism. Compare with the other method, applying linear programming in this Dobb Douglas is more efficient with respect to cost, capital, and the ability to analyze the result of data [4].

3. Research Method

This research variables were workforce (with days, people and time work as units), farm area (ha), fertilizer, and seeds in metric unit (kg) while tools, pesticides, and capitals are measured by rupiah (IDR) currency.

Primer data through field observation were collected by interviews with local migrant farmers using a questionnaire. Selected respondents was conducted proportionally from 570 questionnaires spreading over the migrant settlement units and obtained 268 questionnaires (268 households), which were assumed as representative.

Linear programming analysis

The general form of linear programming analysis

$$\text{maximizing } Z = \sum_{j=1}^{m} c_j x_j$$

with boundary condition

$$\sum_{i=1}^{n}\sum_{j=1}^{m} a_{ij}x_j \le b_i, \quad x_j \ge 0,$$

where $i = 1, 2, 3, 4, ..., 8$; $j = 1, 2, 3, 4, 5$ and,

$(c_1 - c_5)$ = net income from farming, rice, corns, cashew nuts, beans, cocoas,

$(x_1 - x_2)$ = desired plant pattern scope,

$(a_{1j} - a_{8j})$ = the coefficient of resource usage,

$(b_1 - b_8)$ = limited economic resource (land, fertilizer, seeds, pesticide, tools, labour, meds, capital),

Z = the optimum income of local migrant.

Food crops

x_1 = rice field (kg/ha),

x_2 = peanut (kg/ha),

x_3 = corn (kg/ha).

Plantation crops

x_4 = cashew nut (kg/ha),

x_5 = cocoa (kg/ha).

4. Results and Discussion

The advantage of farming (rice, corns, beans, cashew nuts, and cocoas) is the objective function and the coefficient of the constraint function. The score was obtained using optimum resources analyzed by Cobb Douglas function and constraint boundaries were obtained from the mean of resources usages (rice, corns, beans, cashew nuts and cocoas).

The previous results from LP analysis (real condition) using available resources (land, fertilizer/urea, TSP, KCl, pesticide, seeds and labour at TSU Lombo I, II, III) show that the income per ha IDR 396,877. Using optimal const analysis, the optimum income result 1 was IDR 1,359,099 per 0.822 ha corns and 0.123 ha cashew nuts. The optimum income at condition 1 (see Table 1) was an increase up to 29.20% from the previous income. At condition 2, by examining the allowed increase objective coefficient range (AI-OCR), the objective function coefficient was changed from the sum of initial coefficient to the current increase as the increasing maximum level value. Maximum income at condition 2 was IDR 2,579,642, which obtained from 0.734 ha land usage and 0.236 ha cashew nuts, so income optimum is rising 15.38%. The increasing input and output value for each 15% at Condition 3 showed that the maximum income was IDR 1,092,361 obtained from 0.864 ha farming corn and cashew nuts, 0.136 acres ha raised 36.94%. From these three conditions, the optimum income with condition 2 was the maximum income. These results have been obtained from the works done by UPT Timusu, UPT Bulu Katoang, and UPT Pencong and shown at Table 1.

Table 1. Optimum analysis recapitulation at farming business (rice, corn, nuts, cashew nuts, and cocoa) at local UPT

Local TSU	Optimum condition	Changes	Optimum solution	(*) OFV (IDR)	(**) PIL (%)
Lombok I,II,III TSU	(1) (2) (3)	Actual condition Present condition Price increase on interval Al-OCR Input and output price increase 15%	$0,822X_2+0,123X_4$ $0,743X_2+0,236X_4$ $0,864X_2+0,136X_4$	396877 1359099 2579642 1092361	29.20 15.38 36.94
Bulu Katoang TSU	(1) (2) (3)	Actual condition Present condition Price increase on interval Al-OCR Input and output price increase 15%	$0,244X_4+0,542X_2+0,214X_5$ $0,244X_4+0,542X_2+0,214X_5$ $0,244X_4+0,542X_2+0,214X_5$	483026 1210836 1700052 1092461	39.89 28.41 44.21
Timusu TSU	(1) (2) (3)	Actual condition Present condition Price increase on interval Al-OCR Input and output price increase 15%	$0,089X_1+0,041X_3+0,214X_4$ $0,089X_1+0,041X_3+0,214X_4$ $0,089X_1+0,041X_3+0,214X_4$	270774 1298166 1316836 1107891	22.41 21.60 29.26
Pencong TSU	(1) (2) (3)	Actual condition Present condition Price increase on interval Al-OCR Input and output price increase 15%	$0,778X_1+0,055X_2+0,294X_3+0,138X_5$ $0,778X_1+0,055X_2+0,294X_3+0,138X_5$ $0,778X_1+0,055X_2+0,294X_3+0,138X_5$	463140 1650593 1368687 1003682	28.06 35.84 46.18

Source: Analyses data

OFV: objective function value, PIL: the percentage of increasing level.

5. Conclusion and Suggestions

(a) Resources owned by local migrants in South Sulawesi has not been optimally utilized.

(b) The Cobb Douglas function is used to measure the real production toward the frontier, which is located on the isoquant line, and the income function can be obtained by using the linear programming.

(c) The local governance in each district (kabupaten) needs to pay attention on the local migrant life since the local migrants have arrived in the district; they apparently have less attention from the government.

(d) Donations such as fertilizer, loans and medicines are needed to prevent the migrants leaving the location.

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